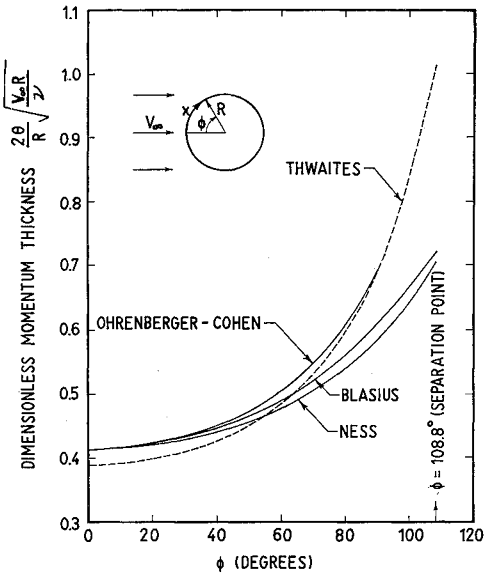

Fig. 1 Comparison with the Blasius series solution of the momentum thickness from several different authors.

dropped the tag on the grouping $u_e\theta^2/\nu_{ref}$ and assumed it to be the dependent variable in the differential equation (2) of Ref. 1.

4) A comparison is made in Fig. 1 of the momentum thickness resulting from several different expressions for the flow of an incompressible fluid around a two-dimensional circular cylinder. The standard for comparison is the Blasius series solution. For this flow, the theoretical potential velocity distribution is $u_e = 2V_\infty \sin\phi$ where V_∞ is the freestream velocity and ϕ is the angle measured from the forward stagnation point; ϕ is related to the distance along the surface x and the cylinder radius R by $\phi = x/R$. For this velocity distribution, the Blasius series solution gives the separation point at $\phi = 108.8°$.

The Blasius curve in Fig. 1 is repeated from Ref. 6; the Ohrenberger-Cohen curve (which is based on a variation of my method) results from the application of Eq. (8), Ref. 2, and is reproduced from Ref. 7; the Thwaites curve is obtained from Ref. 5, whereas the Ness curve is based on Eq. (14) of Ref. 1. For the velocity distribution $u_e = 2V_\infty \sin\phi$, the Thwaites and Ness equations are readily integrated and give, respectively,

$$[(2\theta/R)(V_\infty R/\nu)^{1/2}]_{\text{Thwaites}} = (0.949/\sin^3\phi)[\tfrac{8}{15} - \tfrac{4}{15}(\cos\phi)(\sin^2\phi + 2) - \tfrac{1}{5}\sin^4\phi\cos\phi]^{1/2} \quad (9)$$

$$[(2\theta/R)(V_\infty R/\nu)^{1/2}]_{\text{Ness}} = 0.584/(1 + \cos\phi)^{1/2} \quad (10)$$

The Thwaites result is included to provide a comparison between the method of curve-fitting the results of similar solutions and the new method proposed in Ref. 1. The Thwaites curve does not satisfy the exact value at the stagnation point, whereas the curves obtained by the new method do. (This is a built-in feature in the new method in that the exact value at $x = 0$ is always obtained, regardless of the geometry, for incompressible or compressible flow.) The Thwaites curve and the Ohrenberger-Cohen curve (which they terminated at $\phi = 90°$ in Ref. 7) show wide divergence from the Blasius series solution at high angles of ϕ. The Ness curve, however, over the complete range from the stagnation point to the separation point, does not vary more than 4% (or 5%†) from the Blasius result.

† Because of the small size of Fig. 12.8 of Ref. 6, the coordinates of the Blasius curve are not known as accurately as the coordinates of the other three curves.

It appears, therefore, that the new method gives good results, at least for incompressible flow. The validity of this method for the more general case of a compressible fluid with heat transfer awaits confirmation.

References

[1] Ness, N., "Some comments on the laminar compressible boundary-layer analysis with arbitrary pressure gradient," AIAA J. **5**, 330–331 (1967).

[2] Ohrenberger, J. T. and Cohen, C. B., "Comments on a proposed variation of the Cohen-Reshotko method in boundary-layer theory," AIAA J. **5**, 383–384 (1967).

[3] Thwaites, B., "Approximate calculation of the laminar boundary layer," Aeronaut. Quart. **1**, 245–280 (1949).

[4] Cohen, C. B. and Reshotko, E., "The compressible laminar boundary layer with heat transfer and arbitrary pressure gradient," NACA Rept. 1294 (1956).

[5] Rosenhead, L. (ed.), *Laminar Boundary Layers* (Oxford University Press, London, 1963), Chap. VI, p. 305, Eq. 182.

[6] Schlichting, H., *Boundary Layer Theory* (McGraw-Hill Book Company Inc., New York, 1960), 4th ed., Chap. XII, p. 252, Fig. 12.8.

[7] Ohrenberger, J. T. and Cohen, C. B., private communication (June 30, 1966).

Commenter's Reply to N. Ness

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We have reviewed the reply presented in this issue by N. Ness, defending his method[1] for determining boundary-layer momentum thickness, and we believe that our original criticisms[2] remain valid and that the suggested modification to his method is required. Specific comments on the four points discussed in his reply follow:

1. Ness, in his reply, has simply recast his original result for θ^2 [Eq. (14) of Ref. 1] in terms of β and M_e. Since this does not basically change the result, the fact still holds that the effects of the variation of β, $T_w/T_{e\text{stag}}$, and M_e along the body are only partially accounted for in the Ness method (as demonstrated in Ref. 2). Reference 2 shows that the factor $[\theta_{tr}(x)/\theta_{tr}(0)]$, not included by Ness, can be important. Under the local similarity assumption, this factor also depends on β, $T_w/T_{e\text{stag}}$, and M_e.

2. We cited in Ref. 2 the example of the blunted wedge in supersonic flow as indication that Ness' result for the momentum thickness failed by itself to approach the proper limit far back on the blunted wedge. Ness replies that this is because Eq. (14) of Ref. 1 for θ^2 is for blunt-nosed bodies (rather than sharp-nosed bodies), and hence "should not reduce to the flat-plate case." This explanation seems to miss the main point of our argument, which is the fact that bluntness effects far from the stagnation point of a blunt wedge are negligible, and, hence, in that region $\theta(x)$ increases at the same rate as on a flat plate. The conclusion is that Ness' result is in error by the ratio $[\theta_{tr}(x)/\theta_{tr}(0)]$ in this limit.

3. Ness justifies his chosen form for F [Eq. (12) of Ref. 1] by noting that it is correct for the case of a similarity boundary layer, and furthermore that "previous investigators" have taken relations between parameters which are only strictly valid for the similarity case and assumed them to be valid in the nonsimilar case as well (the method of Thwaites, for example). We suggest that the success of this procedure

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is not universal but depends on the relation which is being generalized. Ness' assumed expression for F is independent of the variation of θ_{tr} along the surface (since θ_{tr} is constant in the similarity case). On the other hand, the constant b in the Thwaites method [see Eq. (8) of Ness' reply] does depend on $d\theta_{tr}/d\beta$ (as can readily be shown) and, therefore, in the nonsimilar case, a variation of θ_{tr} along the surface is accounted for by Thwaites since β varies. Thus, Ness' method fails because he has generalized on a relation valid for similarity, which does not contain parameters that can be important in nonsimilar flow, such as $d\theta_{tr}/d\beta$.

4. With regard to Ness' example to show that his method "gives good results, at least, for incompressible flow," we repeat: What is essential to us is that any method which claims to be general must include all effects that can be shown to be important in limiting cases. It is impressive that his method checks the Blasius calculation so well, but it still errs by a large factor in the case discussed in paragraph 2.

In conclusion, it appears inherent in the Ness method that it can never properly predict the limiting behavior of a flow that is developing toward a terminal similarity different from its initial similar behavior. This conclusion does not apply if the modification proposed by the writers[2] is included. What Ness has not made clear is: When, except for the Blasius calculation cited, is his original method the more accurate?

References

[1] Ness, N., "Some comments on the laminar compressible boundary-layer analysis with arbitrary pressure gradient," AIAA J. 5, 330–331 (1967).

[2] Ohrenberger, J. T. and Cohen, C. B., "Comments on a proposed variation of the Cohen-Reshotko method in boundary layer theory," AIAA J. 5, 383–384 (1967).

Comments on "Effects of Energy Dissipation on a Spinning Satellite"

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THE author's approach[1] to this difficult problem is very interesting. However, one wonders if it is possible to affirm as they make their conclusion: "So far as stability is concerned, one may thus expect to have considerable latitude in choosing the size of the inner body."

In order to study the stability, they conserve only a part of the solution of Eqs. (7) and (8). When we take into account the neglected part, Eqs. (3–6) become differential equations with the coefficients function of the variable τ. A more complete study would show that despite this the system remains stable.

Evidently, the amplitude of the neglected terms tends towards zero exponentially, and consequently we could say that after a sufficient length of time their influence is negligible. However, it is possible that at this moment some of the angles determining the orientation of the satellite take values sufficiently large that it is no longer possible to replace the equations of motion by their linear forms [Eqs. (3–6)].

It is possible to obtain the complete solution of Eqs. (7) and (8). These equations can be written in the following form:

$$MX'' + BX' + KX = 0$$

with

$$X = \begin{vmatrix} \theta_3 \\ \psi_3 \end{vmatrix} \qquad M = \begin{vmatrix} (K+1) & 0 \\ 0 & K'+1Q \end{vmatrix}$$

$$B = \Delta Q \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \qquad K = pQ \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$$

The form of the matrices M, B, and K shows that it is possible to diagonalize them. The eigenvalues are the roots of the characteristic equation,

$$\begin{bmatrix} [1-(K+1)\lambda] & -1 \\ -1 & [1-(K'+1)Q\lambda] \end{bmatrix} =$$
$$(K+1)(K'+1)Q\lambda^2 - [(K+1)+(K'+1)Q]\lambda = 0$$

$$\lambda_1 = 0 \qquad \text{and} \qquad \lambda_2 = \frac{(K+1)+(K'+1)Q}{(K+1)(K'+1)}$$

The corresponding eigenvectors are, respectively,

$$Z_1 = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad \text{and} \qquad Z_2 = \begin{vmatrix} (K'+1)Q \\ -(K+1) \end{vmatrix}$$

The matrix of modes is

$$Z = \begin{vmatrix} 1 & (K'+1)Q \\ 1 & -(K+1) \end{vmatrix}$$

We can calculate

$$\tilde{Z}MZ = \begin{vmatrix} [(K+1)+(K'+1)Q] & 0 \\ 0 & (K+1)(K'+1)Q[(K+1)+(K'+1)Q] \end{vmatrix}$$

$$\tilde{Z}BZ = \Delta Q[(K+1)+(K'+1)Q]^2 \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\tilde{Z}KZ = pQ[(K+1)+(K'+1)Q]^2 \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

Changing the variables

$$\begin{vmatrix} \theta_3 \\ \psi_3 \end{vmatrix} = Z \begin{vmatrix} q_1 \\ q_2 \end{vmatrix}$$

leads to the equations

$$\left| \begin{array}{l} q_1'' = 0 \\ (K+1)(K'+1)q_2'' + \Delta[(K+1) + \\ \qquad (K'+1)Q]q_2' + p[(K+1)+(K'+1)Q]q_2 = 0 \end{array} \right.$$

the solution of which is (the index zero indicating the value when $\tau = 0$)

$$\left| \begin{array}{l} q_1 = q_{10} + q_{10}'\tau \\ q_2 = e^{-\lambda\tau}\left[q_{20}\cos\omega_2\tau + \dfrac{q_{20}' + \lambda q_{20}}{\omega_2}\sin\omega_2\tau \right] \end{array} \right.$$

with

$$\omega_2 = \omega_0 \cdot (1-\epsilon^2)^{1/2}$$

$$\omega_0{}^2 = p\,\frac{(K+1)+(K'+1)Q}{(K+1)(K'+1)} = \frac{k}{\Omega^2}\,\frac{I_3+J_3}{I_3J_3}$$

$$\epsilon = \frac{\Delta}{2}\left[\frac{(K+1)+(K'+1)Q}{p(K+1)(K'+1)}\right]^{1/2} = \frac{\delta}{2}\left(\frac{I_3+J_3}{kI_3J_3}\right)^{1/2}$$

$$\lambda = \epsilon\omega_0 = (\delta/2\Omega)\cdot[(I_3+J_3)/I_3J_3]$$

$$\begin{vmatrix} q_{10} \\ q_{20} \end{vmatrix} = Z^{-1}\begin{vmatrix} \theta_{30} \\ \psi_{30} \end{vmatrix} \qquad \begin{vmatrix} q_{10}' \\ q_{20}' \end{vmatrix} = Z^{-1}\begin{vmatrix} \theta_{30}' \\ \psi_{30}' \end{vmatrix}$$

$$Z^{-1} = \frac{1}{I_3+J_3}\begin{vmatrix} I_3 & J_3 \\ I_1 & -I_1 \end{vmatrix}$$

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